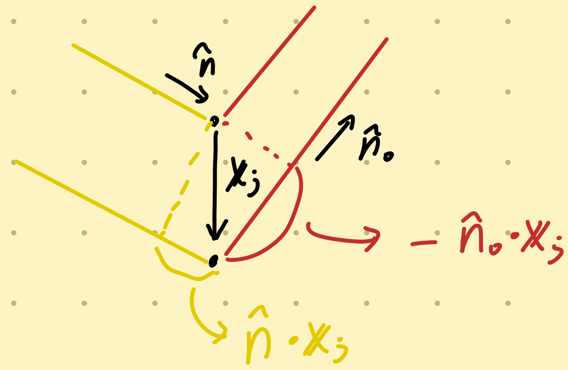
ED 10th week-2

Collections of scatters, 이어서

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{(\text{constant})}\left|\sum_j \left(\hat{\varepsilon}^* \cdot \mathbb{P}_j + (\hat{n}\times\hat{\varepsilon}^*)\cdot\frac{\mathbb{m}}{c}\right)e^{iq\cdot \mathbb{X}_j}\right|^2$$

$$q = k\hat{n} - k\hat{n}_0$$

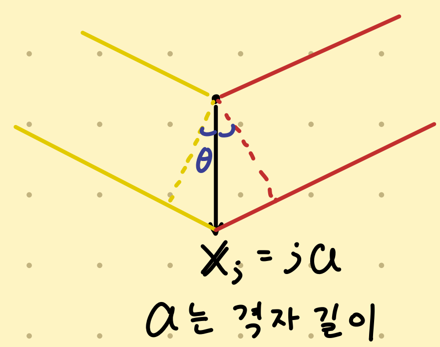
왜 이런 factor를 써야함?



입사각과 산란각을 θ로 같게 둘 때,

$$q \cdot \mathbb{X}_j = \frac{2\pi}{\lambda} \cdot ja \cdot 2\sin\theta = ja\,\frac{4\pi}{\lambda}\sin\theta$$

$$q = \frac{4\pi}{\lambda}\sin\theta$$



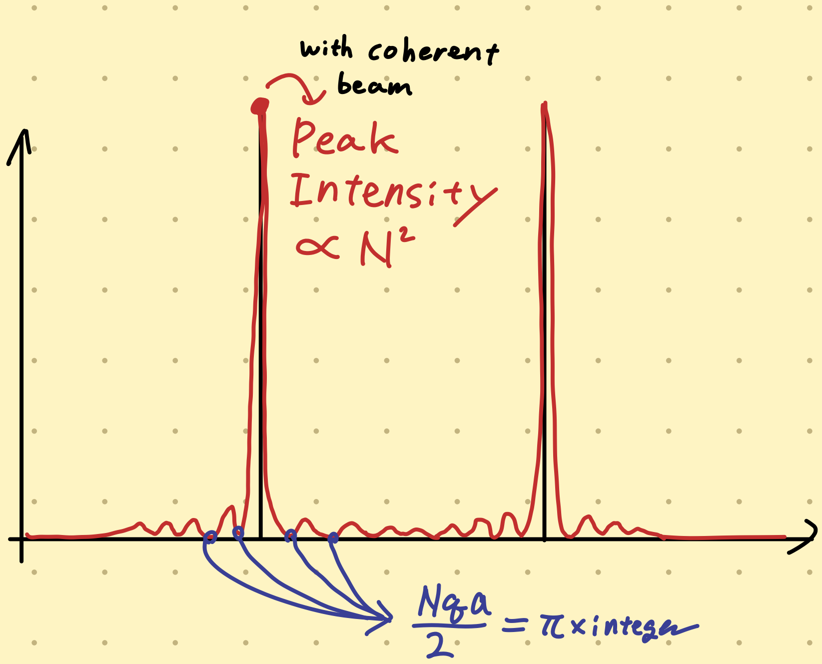
structure factor in 1-D

$$\mathcal{F}(q) = \left|\frac{\sin(Nqa/2)}{\sin(qa/2)}\right|^2$$

Bragg condition

$\frac{qa}{2} = \pi \times \text{integer} \rightarrow \mathcal{F}(q)$ is maximum

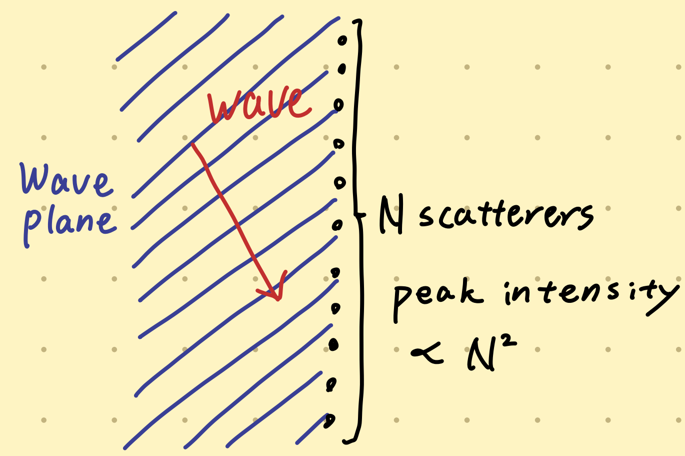
q대입, $\frac{4\pi}{\lambda}\sin\theta = \frac{2\pi}{a} \times \text{integer} \rightarrow 2a\sin\theta = \lambda \cdot \text{integer}$



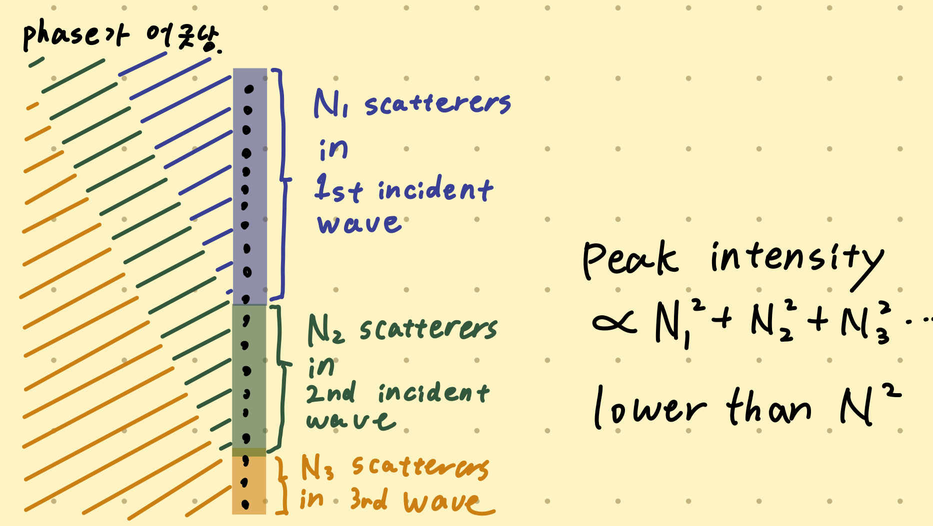
at peak, $\frac{qa}{2} \simeq 0$

$$\mathcal{F}(q) = \left|\frac{\sin(Nqa/2)}{\sin(qa/2)}\right|^2 \simeq \left|\frac{Nqa/2}{qa/2}\right|^2 = N^2$$

지금까지 본 peak intensity $\propto N^2$ 이 나오는 식은 모든 산란자가 같은 phase 를 지닌 평면파에 입사된다 가정함 (coherent incident wave)



⟨coherent state⟩



⟨incoherent state⟩

그러나 산란자에 쪼이는 빛의 phase가 공간에 따라 달라지면,

$$\frac{d\sigma}{d\Omega} = \frac{k^4}{(4\pi\varepsilon_0)^2} \sum_{i}^{\text{Domain number}} \left| \sum_{j}^{N_i} \left[\hat{\varepsilon}^* \cdot p_j + (\hat{n} \times \hat{\varepsilon}^*) \cdot \frac{m_j}{c} \right] e^{i q \cdot x_j} \right|^2$$

이때 N_i 는 i 번째 incident wave를 맞는 입자수.

$$F_{1D}(q) = \sum_{i}^{\text{Domain number}} \left| \frac{\sin(N_i q a/2)}{\sin(N_1 q a)} \right|^2 , \quad \text{Peak intensity} \propto \sum_{i}^{\text{Domain number}} N_i^2$$

이때, $N = \sum_i N_i$ 이므로, 항상 $\sum_i N_i^2 < N^2$ 이다.

결론) coherent 한 빛을 쪼일 때 intensity 가 제일 강해진다.

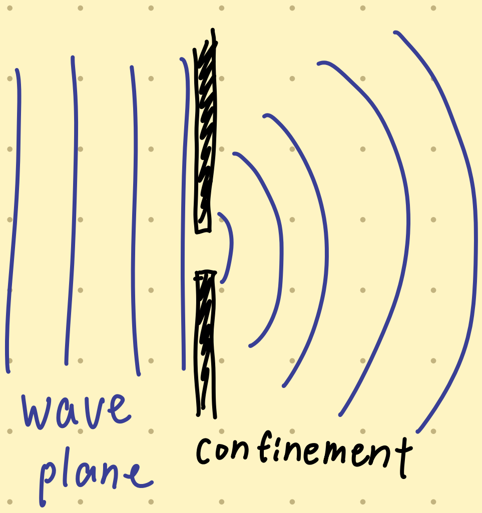
Scalar diffraction theory

Diffraction(회절) : Sommerfeld 가 정의했다.

" Deviation of light rays from rectlinear path.
which cannot be interpreted from reflection or refraction.

교수님은 이 정의가 마음에 안 든다고 한다.

Diffraction is caused by the confinement of lateral extent of the wave



confinement가 wavelength 과 견줄 만한 스케일이거나 그보다 작으면 효과가 있다.

kirhhoff's method

- scalar field
- $(\nabla^2 + k^2)\psi(x) = 0$ inside closed volumn V
- Green function for the Helmholtz equation

$$(\nabla^2 + k^2) G(x, x') = \delta(x - x')$$

- Detour to Green's theorem

$$\boxed{\int (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dx = \oint \left[\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}\right] da}$$

유도과정

$$\int \nabla \cdot (A + A')\, dx = \oint (A + A') \cdot \hat{n}\, da$$

$$A = \phi \nabla \psi \rightarrow \nabla \cdot A = \nabla \cdot (\phi \nabla \psi) = \phi \nabla^2 \psi + (\nabla \phi) \cdot (\nabla \psi)$$

$$A' = \psi \nabla \phi \rightarrow \nabla \cdot A' = \nabla \cdot (\psi \nabla \phi) = \psi \nabla^2 \phi + (\nabla \psi) \cdot (\nabla \phi)$$

$$\nabla \cdot A - \nabla \cdot A' = \phi \nabla^2 \psi - \psi \nabla^2 \phi$$

$$\int A \cdot \hat{n}\, da = \oint \phi (\nabla \psi) \cdot \hat{n}\, da = \oint \phi \frac{\partial \psi}{\partial n} da$$

$$\int A' \cdot \hat{n}\, da = \oint \psi \frac{\partial \phi}{\partial n} da$$

$\phi = G$, $\psi = \psi$ 를 대입해 보자.

$$\nabla^2 G(x, x') = -k^2 G(x, x') + \delta(x - x'), \quad \nabla^2 \psi = -k^2 \psi$$

$$G \nabla^2 \psi - \psi \nabla^2 G = -\psi \delta(x - x')$$

$$\int G \nabla^2 \psi - \psi \nabla^2 G\, dx = -\int \psi(x) \delta(x - x')\, dx = -\psi(x')$$

$$\psi(x') = \oint \psi(x) (\hat{n} \cdot \nabla G(x, x')) - G(x, x') (\hat{n} \cdot \nabla \psi(x))\, da$$

$$\psi(\mathbf{x}') = \oint \psi(\mathbf{x})\left(\hat{n}\cdot\nabla G(\mathbf{x},\mathbf{x}')\right) - G(\mathbf{x},\mathbf{x}')\left(\hat{n}\cdot\nabla\psi(\mathbf{x})\right) da$$

G는 구면파다. $\mathbf{R} = \mathbf{x}-\mathbf{x}'$, $G(\mathbf{x},\mathbf{x}') = \frac{1}{4\pi R} e^{i\mathbf{k}\cdot\mathbf{R}}$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} = \left(\hat{x}\frac{\partial R}{\partial x} + \hat{y}\frac{\partial R}{\partial y} + \hat{z}\frac{\partial R}{\partial z}\right)\frac{\partial}{\partial R} = \frac{\mathbf{R}}{R}\frac{\partial}{\partial R}$$

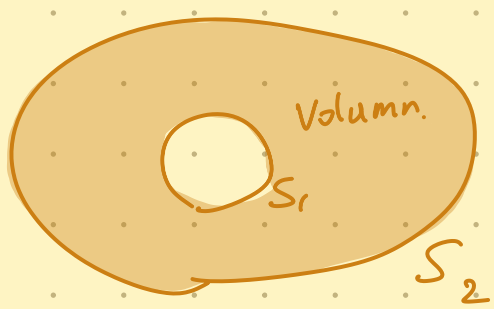
$$\nabla G = \left(\frac{\mathbf{R}}{R}\frac{\partial}{\partial R}\frac{1}{4\pi R}\right)e^{i\mathbf{k}\cdot\mathbf{R}} + \frac{1}{4\pi R}\left\{\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)e^{i\mathbf{k}\cdot\mathbf{R}}\right\}$$

$$= -\frac{\mathbf{R}}{4\pi R^3} e^{i\mathbf{k}\cdot\mathbf{R}} + \frac{i\mathbf{k}}{4\pi R} e^{i\mathbf{k}\cdot\mathbf{R}}$$

$\mathbf{k} = k\cdot\frac{\mathbf{R}}{R}$ 을 대입, $\mathbf{k}\cdot\mathbf{R} = kR$

$$\nabla G = -\frac{\mathbf{R}}{4\pi R^3} e^{ikR} + \frac{ik\mathbf{R}}{4\pi R^2} e^{ikR}$$

$$= -\frac{e^{ikR}}{4\pi R}\left(\frac{\mathbf{R}}{R^2} + \frac{ik\mathbf{R}}{R}\right) = -\frac{e^{ikR}}{4\pi R}\cdot ik\frac{\mathbf{R}}{R}\left(1 + \frac{1}{kR}\right)$$



S_2는 저 멀리 떨어진 곳으로 보내 적분을 0으로 만든다.

키르히호프의 적분 공식 (Jackson 10.79) 내가 유도 과정 중에 부호를 잘못썼나?

$$\psi(\mathbf{x}) = -\frac{1}{4\pi}\int_{S_1}\frac{e^{ikR}}{R}\,\hat{n}'\cdot\left[\nabla'\psi + ik\left(1+\frac{i}{kR}\right)\frac{\mathbf{R}}{R}\psi(\mathbf{x}')\right]da'$$

즉, S_1에서 ψ와 $\nabla\psi$ 값을 알아야 한다.

Usually values are not known

approximation.

- 벽으로 막힌 데에서는 $\nabla\psi$ 와 ψ 값이 0 이다.
- 벽이 없는 곳에서는 incident field 값과 같다.

→ 수학적으로 별로인 approximation.