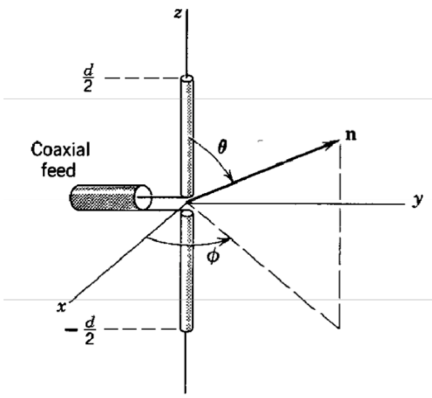
HW4 Due by 6:00 PM 05/29. Web uploading.

1. (20) Radiation from a center-fed linear antenna



Consider a coaxial center-fed antenna with the current density,

$$\vec{J}(\vec{x}) = I \sin\left(\frac{kd}{2} - k|z|\right)\delta(x)\delta(y)\hat{z}$$
$$|z| < d/2$$

(a) (3) Find the vector potential in the radiation zone

(b) (3) Find the H and E fields

(c) (3) Show that the time-average power per solid angle can be obtained as

$$\frac{dP}{d\Omega} = \frac{Z_0 I^2}{8\pi^2}\left|\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)\right|^2 / |\sin\theta|^2$$

(d) (3) Show that it reduces to the dipole radiated power, $\frac{dP}{d\Omega} = \frac{Z_0 I^2}{8128}(kd)^2|\sin\theta|^2$, in the long wavelength limit

(e) (4) Find the time-average radiated power per solid angle for the special case of $kd = \pi$ and $kd = 2\pi$,

(f) (4) Plot the angular distributions for (e).

2. (20) Angular distribution of the multipole radiation

The angular distribution of a pure multipole of order (l,m) can be obtained as

$$\frac{dP(l,m)}{d\Omega} = \frac{Z_0}{2k^2}|a(l,m)|^2|X_{lm}|^2,$$

with the X_{lm} defined in the textbook.

(a) (10) Find the explicit form of $|X_{lm}(\theta, \varphi)|^2$ for $l = 1$ $(dipole)$ & 2 $(quqdrupole)$ with allowed m's for each case.

(b) (10) Draw the radiation patterns for pure (l,m) multipoles

<u>J.D. Jackson, Chapter 9 (3rd edition)</u>

3. (20) Prob. 9.1

4. (20) Radiating quadrupole (Prob. 9.2)

5. (20) Radiating two fixed dipoles (Prob. 9.15)