1. In one spatial dimension, two frames of reference S and S' have coordinates (x, t) and (x', t') respectively. The coordinates are related by $t' = t$ and

$$x' = f(x, t)$$

Viewed from frame S, a particle follows a trajectory $x = x(t)$. It has velocity $v = \dot{x}$ and acceleration $a = \ddot{x}$. Viewed from S', the trajectory is $x' = f(x(t), t)$. Using the chain rule, show that the speed and acceleration of the particle in S' are given by

$$\begin{aligned}\frac{dx'}{dt'} &= v\frac{\partial f}{\partial x} + \frac{\partial f}{\partial t} \\ \frac{d^2x'}{dt'^2} &= a\frac{\partial f}{\partial x} + v^2\frac{\partial^2 f}{\partial x^2} + 2v\frac{\partial^2 f}{\partial x\,\partial t} + \frac{\partial^2 f}{\partial t^2}.\end{aligned}$$

Suppose now that both S and S' are inertial frames. Explain why the function f must obey $\partial^2 f/\partial x^2 = \partial^2 f/\partial x \partial t = \partial^2 f/\partial t^2 = 0$. What is the most general form of f with these properties? Interpret this result.

5. A particle of mass m, charge q and position $\mathbf{x}$ moves in a constant, uniform magnetic field $\mathbf{B}$ which points in a horizontal direction. The particle is also under the influence of gravity, $\mathbf{g}$, acting vertically downwards. Write down the equation of motion and show that it is invariant under translations $\mathbf{x} \to \mathbf{x} + \mathbf{x}_0$. Obtain

$$\dot{\mathbf{x}} = \alpha \mathbf{x} \times \mathbf{n} + \mathbf{g}t + \mathbf{a}$$

where $\alpha = qB/m$, $\mathbf{n}$ is a unit vector in the direction of $\mathbf{B}$ and $\mathbf{a}$ is a constant vector. Show that, with a suitable choice of origin, $\mathbf{a}$ can be written in the form $\mathbf{a} = a\mathbf{n}$.

By choosing suitable axes, show that the particle undergoes a helical motion with a constant horizontal drift.

Suppose that you now wish to eliminate the drift by imposing a uniform electric field $\mathbf{E}$. Determine the direction and magnitude of $\mathbf{E}$.

1. In a system of particles, the ith particle has mass m_i and position vector $\mathbf{x}_i$ with respect to a fixed origin. The centre of mass of the system is at $\mathbf{R}$. Show that $\mathbf{L}$, the total angular momentum of the system about the origin, and $\mathbf{L}_{\mathrm{CoM}}$, the total angular momentum of the system about the centre of mass, are related by

$$\mathbf{L}_{CoM} = \mathbf{L} - \mathbf{R} \times \mathbf{P}$$

where $\mathbf{P}$ is the total linear momentum of the system.

Given that $d\mathbf{P}/dt = \mathbf{F}$ where $\mathbf{F}$ is the total external force and $d\mathbf{L}/dt = \boldsymbol{\tau}$ where $\boldsymbol{\tau}$ is the total external torque about the origin, show that

$$\frac{d\mathbf{L}_{CoM}}{dt} = \boldsymbol{\tau}_{CoM},$$

where $\boldsymbol{\tau}_{CoM}$ is the total external torque about the centre of mass.