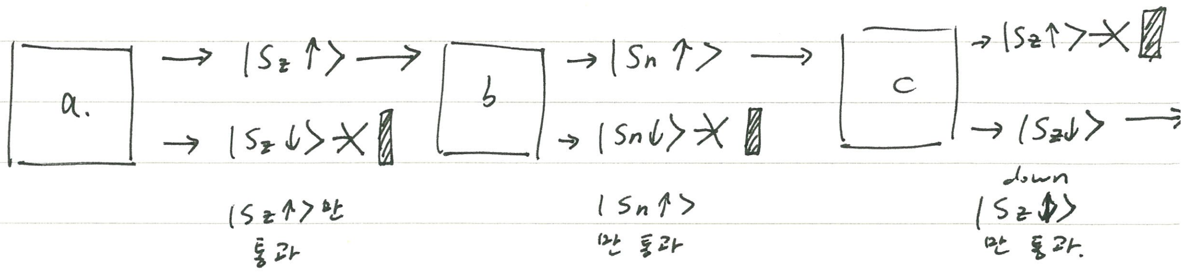
QM week 3 HW

20262182 김선지

Sakurai P 1.15.



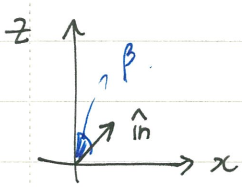
맨 마지막 $|S_z\downarrow\rangle$의 세기는 얼마인가?

풀이

$$|S_z\downarrow\rangle \text{의 세기} = |\langle S_z\downarrow|S_n\uparrow\rangle|^2\,|\langle S_n\uparrow|S_z\uparrow\rangle|^2$$

$|S_n\uparrow\rangle$를 $|S_z\uparrow\rangle$과 $|S_z\downarrow\rangle$의 basis로 나타내는 게 관건.

먼저 $\mathbf{S}\cdot\hat{n}$를 정의해야 함.



$$\hat{n} = \hat{x}\sin\beta + \hat{z}\cos\beta.$$

$$\mathbf{S} = \hat{x}S_x + \hat{y}S_y + \hat{z}S_z$$

$$\mathbf{S}\cdot\hat{n} = \sin\beta\, S_x + \cos\beta\, S_z = \frac{\hbar}{2}\sin\beta\begin{pmatrix}0&1\\1&0\end{pmatrix} + \frac{\hbar}{2}\cos\beta\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\mathbf{S}\cdot\hat{n} = \frac{\hbar}{2}\begin{pmatrix}\cos\beta & \sin\beta\\ \sin\beta & -\cos\beta\end{pmatrix}$$

이것에 대해 $\frac{\hbar}{2}$의 eigenvalue를 가지는 eigenket을 찾아라. $|S_n\uparrow\rangle$를 찾아라.

$$|S_n\uparrow\rangle = \begin{pmatrix}a\\b\end{pmatrix}$$ 일 때, $$\begin{pmatrix}\cos\beta & \sin\beta\\ \sin\beta & -\cos\beta\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

normalization condition을 이용하면,

$$\left\{\begin{aligned} &a\cos\beta + b\sin\beta = a\\ &a\sin\beta - b\cos\beta = b\\ &a^2+b^2=1\end{aligned}\right. \rightarrow \left\{\begin{aligned} &ab\cos\beta + b^2\sin\beta = ab\\ &a^2\sin\beta - ab\cos\beta = ab\end{aligned}\right.$$

$$(a^2+b^2)\sin\beta = 2ab.$$

$$\sin\beta = 2ab.$$

$$b^2 = \frac{1}{2}(1-\cos\beta) = \frac{1}{2}\left(\sin^2\frac{\beta}{2} + \cos^2\frac{\beta}{2} - \cos^2\frac{\beta}{2} + \sin^2\frac{\beta}{2}\right)$$

$$b^2 = \sin^2\frac{\beta}{2},\quad b = \sin\frac{\beta}{2} \qquad a = \cos\frac{\beta}{2}$$

$$\therefore |S_n\uparrow\rangle = \begin{pmatrix}\cos\frac{\beta}{2}\\ \sin\frac{\beta}{2}\end{pmatrix} = \cos\frac{\beta}{2}|S_z\uparrow\rangle + \sin\frac{\beta}{2}|S_z\downarrow\rangle$$

$$|\langle S_z\downarrow|S_n\uparrow\rangle|^2\,|\langle S_n\uparrow|S_z\uparrow\rangle|^2 = \left|\cos\frac{\beta}{2}\right|^2\left|\sin\frac{\beta}{2}\right|^2 = \left(\frac{1}{2}\sin\beta\right)^2 = \frac{1}{4}\sin^2\beta$$

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20262182 김신리

Sakura: 1.21.

S_z+ state에 대해 $\langle (\Delta S_x)^2 \rangle \equiv \langle S_x^2 \rangle - \langle S_x \rangle^2$ 를 구하라.

$A \to S_x$ $B \to S_y$ 일 때

$\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \frac{1}{4} \left| \langle [A, B] \rangle \right|^2$ 를 확인하라.

풀이 S_z+ state를 $|\uparrow\rangle$ 라고, S_z- state를 $|\downarrow\rangle$ 라고 표기.

$$S_x = \frac{\hbar}{2} |\downarrow\rangle\langle\uparrow| + \frac{\hbar}{2} |\uparrow\rangle\langle\downarrow|$$

$$S_x^2 = \frac{\hbar^2}{4} \left(|\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| \right)\left(|\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow| \right)$$

$$= \frac{\hbar^2}{4} \left(\cancel{|\downarrow\rangle\langle\uparrow|\downarrow\rangle\langle\uparrow|} + |\downarrow\rangle\langle\uparrow|\uparrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\downarrow|\downarrow\rangle\langle\uparrow| + \cancel{|\uparrow\rangle\langle\downarrow|\uparrow\rangle\langle\downarrow|} \right.$$

$$= \frac{\hbar^2}{4} |\downarrow\rangle\langle\downarrow| + \frac{\hbar^2}{4} |\uparrow\rangle\langle\uparrow|$$

$$\langle S_x \rangle = \langle\uparrow| S_x |\uparrow\rangle = \frac{\hbar}{2} \left\{ \langle\uparrow|\downarrow\rangle\langle\uparrow|\uparrow\rangle + \langle\uparrow|\uparrow\rangle\langle\downarrow|\uparrow\rangle \right\} = 0.$$

$$\langle S_x^2 \rangle = \langle\uparrow| S_x^2 |\uparrow\rangle = \frac{\hbar^2}{4}$$

$$\langle (\Delta S_x)^2 \rangle = \frac{\hbar^2}{4}$$

S_y 같은 경우는 어떨까?

$$S_y = -i\frac{\hbar}{2} |\uparrow\rangle\langle\downarrow| + i\frac{\hbar}{2} |\downarrow\rangle\langle\uparrow|$$

$$S_y^2 = \frac{\hbar^2}{4} |\uparrow\rangle\langle\uparrow| + \frac{\hbar^2}{4} |\downarrow\rangle\langle\downarrow|$$

$$\langle S_y \rangle = 0$$

$$\langle S_y^2 \rangle = \frac{\hbar^2}{4}$$

$$[S_x, S_y] = i\hbar S_z$$

$$\langle [S_x, S_y] \rangle = i\hbar \langle S_z \rangle = i\frac{\hbar^2}{2}$$

$$\left| \langle [S_x, S_y] \rangle \right|^2 = \frac{\hbar^4}{4}$$

$$\langle (\Delta S_x)^2 \rangle \langle (\Delta S_y)^2 \rangle \geq \frac{1}{4} \left| \langle [S_x, S_y] \rangle \right|^2$$

$$\frac{\hbar^2}{4} \cdot \frac{\hbar^2}{4} \geq \frac{1}{4} \cdot \frac{\hbar^4}{4}.$$

등식을 만족한다.