QM. Week 5 HW . 2026182 김신지

Sakurai. 2.3 .

a. 전자가 $S_x = \frac{\hbar}{2}$ state 에 있을 확률을 구하라.

풀이 →

먼저 notation 과 기본적인 값들을 정리하자.

S_z 의 eigen ket 을 $|\uparrow\rangle$, $|\downarrow\rangle$ 라고 표기.

S_x 의 eigen ket 을 $|\uparrow x\rangle$, $|\downarrow x\rangle$ 라고 표기.

$S_n = \mathbf{S}\cdot\hat{n}$ 의 eigen ket 을 $|\uparrow n\rangle$, $|\downarrow n\rangle$ 라고 표기.

$t=0$ 에서 문제에서 제시한 상태는 $|\uparrow n\rangle$ 이다.

t 시간이 지난 후 이 상태는 $|\uparrow n(t)\rangle$ 라 표기.

전자의 해밀토니안 (자기장 속) 은

$$\mathcal{H} = -\frac{e}{mc}\mathbf{B}\cdot\mathbf{S} = -\gamma\mathbf{B}\cdot\mathbf{S} \quad , \quad \gamma = \frac{e}{mc} .$$

진짜 풀이 →

$$\hat{n} = \hat{x}\sin\beta + \hat{z}\cos\beta \quad , \quad \mathbf{S} = \hat{x}S_x + \hat{y}S_y + \hat{z}S_z$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{n}\cdot\mathbf{S} = \sin\beta\, S_x + \cos\beta\, S_z = \frac{\hbar}{2}\begin{pmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{pmatrix} .$$

$|\uparrow n\rangle$ 을 구해 보자.

$$|\uparrow n\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \qquad a, b \in \mathbb{C} \; , \quad \cancel{a^2+b^2} \quad |a|^2 + |b|^2 = 1$$ 이라 두면,

$$S_n|\uparrow n\rangle = \frac{\hbar}{2}\begin{pmatrix} a\cos\beta & b\sin\beta \\ a\sin\beta & -b\cos\beta \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ b \end{pmatrix} .$$

$$\begin{cases} a\cos\beta + b\sin\beta = a \\ a\sin\beta - b\cos\beta = b . \end{cases}$$

$$\longrightarrow \frac{a}{b} = \frac{\sin\beta}{1-\cos\beta} = \frac{2\cos\frac{\beta}{2}\sin\frac{\beta}{2}}{(\cos^2\beta/2 + \sin^2\beta/2) - (\cos^2\beta/2 - \sin^2\beta/2)}$$

$$\longrightarrow \frac{a}{b} = \frac{1+\cos\beta}{\sin\beta} = \frac{(\cos^2\beta/2 + \sin^2\beta/2) + (\cos^2\beta/2 - \sin^2\beta/2)}{2\cos\frac{\beta}{2}\sin\frac{\beta}{2}} .$$

두 식 모두 정리하면 $\frac{a}{b} = \frac{\cos\frac{\beta}{2}}{\sin\frac{\beta}{2}}$ 가 나온다. 따라서,

$$|\uparrow n\rangle = \begin{pmatrix} \cos\beta/2 \\ \sin\beta/2 \end{pmatrix} = \cos\frac{\beta}{2}|\uparrow\rangle + \sin\frac{\beta}{2}|\downarrow\rangle$$

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해밀토니안을 구하면,

$$\mathcal{H} = -\gamma \mathbf{S}\cdot\mathbf{B} = -\gamma \mathbf{S}\cdot(B_0 \hat{z}) = -\gamma B_0 S_z = -\frac{\hbar\gamma B_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

$|\uparrow\rangle$ 와 $|\downarrow\rangle$ 는 $\mathcal{H}$ 에 대한 eigen state.

$$\mathcal{H}|\uparrow\rangle = -\frac{\hbar\gamma B_0}{2}|\uparrow\rangle = E_\uparrow|\uparrow\rangle, \quad \mathcal{H}|\downarrow\rangle = \frac{\hbar\gamma B_0}{2}|\downarrow\rangle = -E_\uparrow|\downarrow\rangle$$

$|\uparrow(t)\rangle$ 와 $|\downarrow(t)\rangle$ 는 $|\uparrow\rangle$ 와 $|\downarrow\rangle$ 에서 시간에 따라 phase만 바뀜.

$$|\uparrow(t)\rangle = \exp\left(-i\frac{E_\uparrow}{\hbar}t\right)|\uparrow\rangle = \exp\left(i\frac{\gamma B_0}{2}\right)|\uparrow\rangle$$
$$|\downarrow(t)\rangle = \exp\left(i\frac{E_\uparrow}{\hbar}t\right)|\downarrow\rangle = \exp\left(-i\frac{\gamma B_0}{2}\right)|\downarrow\rangle$$

$$|\uparrow_n(t)\rangle = \cos\left(\frac{\beta}{2}\right)|\uparrow(t)\rangle + \sin\frac{\beta}{2}|\downarrow(t)\rangle$$
$$= \cos\frac{\beta}{2}\exp\left(i\frac{\gamma B_0}{2}t\right)|\uparrow\rangle + \sin\frac{\beta}{2}\exp\left(-i\frac{\gamma B_0}{2}t\right)|\downarrow\rangle$$

$$\langle\uparrow_x|\uparrow_n(t)\rangle = \frac{1}{\sqrt{2}}\left[\langle\uparrow|\uparrow_n(t)\rangle + \langle\downarrow|\uparrow_n(t)\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[\cos\beta/2\,\exp\left(i\frac{\gamma B_0}{2}t\right) + \sin\beta/2\,\exp\left(-i\frac{\gamma B_0}{2}t\right)\right]$$

$$|\langle\uparrow_x|\uparrow_n(t)\rangle|^2 = \frac{1}{2}\left[\cos\beta/2\,\exp\left(i\frac{\gamma B_0}{2}t\right) + \sin\frac{\beta}{2}\exp\left(-i\frac{\gamma B_0}{2}t\right)\right]\left[\cos\frac{\beta}{2}\exp\left(-i\frac{\gamma B_0}{2}t\right) + \sin\frac{\beta}{2}\exp\left(i\frac{\gamma B_0}{2}t\right)\right]$$

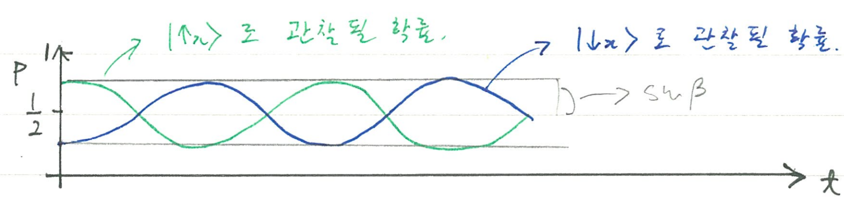
$$= \frac{1}{2}\left[\cos^2\frac{\beta}{2} + \sin^2\frac{\beta}{2} + \sin\frac{\beta}{2}\cos\frac{\beta}{2}\left(\exp(-i\gamma B_0 t) + \exp(i\gamma B_0 t)\right)\right]$$
$$= \frac{1}{2}\left[1 + 2\cos(\gamma B_0 t)\sin\frac{\beta}{2}\cos\frac{\beta}{2}\right]$$

$|\langle\uparrow_x|\uparrow_n(t)\rangle|^2 = \frac{1}{2}\left[1 + \cos(\gamma B_0 t)\sin\beta\right]$ 이것이 t 시간 뒤에 $|\uparrow_x\rangle$ 상태로 발견할 확률.

b. S_x 의 기댓값을 시간에 대한 함수로 구하라.

→ 풀이

$$|\langle\downarrow_x|\uparrow_n(t)\rangle|^2 = 1 - |\langle\uparrow_x|\uparrow_n(t)\rangle|^2$$
$$= \frac{1}{2}\left[1 - \cos(\gamma B_0 t)\sin\beta\right]$$



$$\langle S_x\rangle_{(t)} = \frac{\hbar}{2}\left|\langle\uparrow_x|\uparrow_n(t)\rangle\right|^2 - \frac{\hbar}{2}\left|\langle\downarrow_x|\uparrow_n(t)\rangle\right|^2$$
$$= \frac{\hbar}{2}\cdot\frac{1}{2}\cdot 2\cos(\gamma B_0 t)\sin\beta$$
$$= \frac{\hbar}{2}\cos(\gamma B_0 t)\sin\beta$$

C. $\beta \to 0$ 의 극한과 $\beta \to \frac{\pi}{2}$ 의 극한에서 결과를 정당화하라.

풀이 →

(i) $\beta \to 0$ 이면, $n = \hat{z}$, $|\uparrow_n\rangle = |\uparrow\rangle$

$|\uparrow_n\rangle$은 phase 만 달라질 뿐 계속 $|\uparrow\rangle$ 상태로 남아있다.

$|\langle\uparrow_x|\uparrow\rangle| = \frac{1}{2}$, $|\langle\downarrow_x|\uparrow\rangle| = \frac{1}{2}$ 이므로,

$|\uparrow_n\rangle$은 ~~항상~~ $|\uparrow_x\rangle$으로 발견될 확률이 항상 $\frac{1}{2}$, $|\downarrow_x\rangle$의 경우도 마찬가지이다.

$\sin\beta = 0$ 이므로, $\langle S_x\rangle = 0$ 인 게 정당하다.

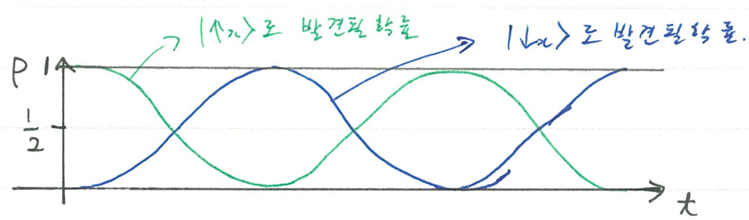
(ii) $\beta \to \frac{\pi}{2}$ 이면, $n = \hat{x}$, $|\uparrow_n\rangle = |\uparrow_x\rangle$

$\langle S_x\rangle(0) = \frac{\hbar}{2}\sin\beta = \frac{\hbar}{2}$ 로, $|\uparrow_x\rangle$의 $\langle S_x\rangle$와 같다.

스핀이 x-y 평면에서 계속 돌기 때문에,

$|\uparrow_x\rangle$로 발견될 확률은 $\frac{1}{2}(1+\cos(\gamma B_0 t))$,

$|\downarrow_x\rangle$로 발견될 확률은 $\frac{1}{2}(1-\cos(\gamma B_0 t))$ 이다.



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Sakurai 2.12

$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}} e^{i\delta}|1\rangle$ $\delta \in \mathbb{R}$

a. $\langle x'|\alpha, t\rangle$ 를 찾아라. $\langle x\rangle$ 와 $\langle p\rangle$ 를 찾아라.

→ 풀이

$$|0(t)\rangle = \exp\left(-i\frac{E_0}{\hbar}t\right)|0\rangle = \exp\left(-i\frac{1}{2}\omega t\right)|0\rangle$$

$$|1(t)\rangle = \exp\left(-i\frac{E_1}{\hbar}t\right)|1\rangle = \exp\left(-i\frac{3}{2}\omega t\right)|1\rangle$$

$$|\alpha(t)\rangle = \frac{1}{\sqrt{2}}\exp\left(-i\frac{1}{2}\omega t\right)|0\rangle + \frac{1}{\sqrt{2}}\exp\left(-i\frac{3}{2}\omega t + i\delta\right)|1\rangle$$

먼저 $\psi_0(x') = \langle x'|0\rangle$ 와 $\psi_1(x') = \langle x'|1\rangle$ 를 구해보자.

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{1}{m\omega}p\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - i\frac{1}{m\omega}p\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) \qquad p = i\sqrt{\frac{m\hbar\omega}{2}}(a^\dagger - a), \qquad p = -i\hbar\frac{\partial}{\partial x}$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x' + \frac{\hbar}{m\omega}\frac{\partial}{\partial x'}\right), \quad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x' - \frac{\hbar}{m\omega}\frac{\partial}{\partial x'}\right)$$

$$\langle x'|a|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\langle x'|x + \frac{\hbar}{m\omega}\frac{\partial}{\partial x}|0\rangle$$
$$= \sqrt{\frac{m\omega}{2\hbar}}\left(x' + \frac{\hbar}{m\omega}\frac{\partial}{\partial x'}\right)\psi_0(x') = 0$$

$$x'\psi_0 + \frac{\hbar}{m\omega}\frac{\partial}{\partial x'}\psi_0 = 0, \quad x_0^2 = \frac{\hbar}{m\omega}, \quad x'\psi_0 + x_0^2\frac{\partial}{\partial x'}\psi_0 = 0$$

$$\frac{1}{\psi_0}d\psi_0 = -\frac{1}{x_0^2}x'dx', \quad \psi_0(x') = C\exp\left(-\frac{x'^2}{2x_0^2}\right)$$

normalization으로 C를 결정. $|C|^2\int_{-\infty}^{\infty}dx' \exp\left(-\frac{x'^2}{x_0^2}\right) = |C|^2 x_0\sqrt{\pi} = 1.$

$$C = \frac{1}{\pi^{1/4}x_0^{\frac{1}{2}}} \qquad \psi_0(x') = \frac{1}{\pi^{1/4}x_0^{1/2}}\exp\left(-\frac{x'^2}{2x_0^2}\right)$$

이제 $\psi_1(x')$ 를 계산.

$$\psi_1(x') = \langle x'|a^\dagger|0\rangle = \frac{1}{x_0\sqrt{2}}\langle x'|x - x_0^2\frac{\partial}{\partial x}|0\rangle = \frac{1}{x_0\sqrt{2}}\left(x' - x_0^2\frac{\partial}{\partial x'}\right)\psi_0(x')$$

$$\frac{\partial}{\partial x'}\psi_0(x') = -\frac{x'}{x_0^2}\psi_0(x'), \quad \psi_1(x') = \frac{1}{x_0\sqrt{2}}(x' + x')\psi_0 = \frac{\sqrt{2}}{x_0}x'\psi_0$$

$$\therefore \psi_1(x') = \frac{1}{\pi^{1/4}x_0^{3/2}}\sqrt{2}\,x'\exp\left(-\frac{x'^2}{2x_0^2}\right)$$

$$\langle x'|\alpha, t\rangle = \frac{1}{\sqrt{2}}\exp\left(-i\frac{1}{2}\omega t\right)\psi_0 + \frac{1}{\sqrt{2}}\exp\left(-i\frac{3}{2}\omega t + i\delta\right)\psi_1$$
$$= \frac{1}{\sqrt{2}}\left(\exp\left(-i\frac{1}{2}\omega t\right) + \frac{x'}{x_0}\sqrt{2}\exp\left(-i\frac{3}{2}\omega t + i\delta\right)\right)\psi_0$$

정리) $\psi_0(x') = \frac{1}{\pi^{1/4}\sqrt{x_0}} \exp\left(-\frac{x'^2}{2x_0^2}\right)$, $\psi_1(x') = \frac{\sqrt{2}}{x_0} x' \psi_0$

$$\frac{\partial \psi_0}{\partial x'} = -\frac{x'}{x_0^2}\psi_0 , \quad \frac{\partial \psi_1}{\partial x'} = \frac{\sqrt{2}}{x_0}\left(1 - \frac{x'^2}{x_0^2}\right)\psi_0$$

$$\int x'^2 \psi_0^* \psi_0 \, dx' = \frac{1}{x_0\sqrt{\pi}} \int x'^2 \exp\left(-\frac{x'^2}{x_0^2}\right) dx' = \frac{1}{x_0\sqrt{\pi}} \cdot \frac{1}{2} x_0^3 \sqrt{\pi} = \frac{x_0^2}{2}$$

$$\psi_\alpha(x',t) = \left[\frac{1}{\sqrt{2}} \exp\left(-i\frac{1}{2}\omega t\right) + \frac{x'}{x_0}\exp\left(-i\frac{3}{2}\omega t + i\delta\right)\right]\psi_0$$

$$\frac{\partial \psi_\alpha}{\partial x'} = \left[-\frac{x'}{x_0^2\sqrt{2}}\exp\left(-i\frac{\omega}{2}t\right) + \left(\frac{1}{x_0} - \frac{x'^2}{x_0^3}\right)\exp\left(-i\frac{3}{2}\omega t + i\delta\right)\right]\psi_0$$

$$\psi_\alpha^* \psi_\alpha = \left[\frac{1}{2} + \frac{x'^2}{x_0^2} + \frac{x'}{x_0\sqrt{2}}\left\{\exp(-i\omega t + i\delta) + \exp(i\omega t - i\delta)\right\}\right]\psi_0^*\psi_0$$
$$= \left[\frac{1}{2} + \frac{x'^2}{x_0^2} + \frac{x'\sqrt{2}}{x_0}\cos(\omega t - \delta)\right]\psi_0^*\psi_0$$

$$\langle x \rangle_\alpha(t) = \int_{-\infty}^{\infty} x' \psi_\alpha^* \psi_\alpha \, dx' = \int_{-\infty}^{\infty} \frac{\sqrt{2}}{x_0} x'^2 \cos(\omega t - \delta)\psi_0^*\psi_0 \, dx' = \frac{\sqrt{2}}{x_0}\cos(\omega t - \delta)\cdot\frac{x_0^2}{2}$$

기함수는 적분해도 0
우함수 항만 적분

$$\langle x \rangle_\alpha(t) = \frac{x_0}{\sqrt{2}}\cos(\omega t - \delta)$$

$$\psi_\alpha^* \frac{\partial \psi_\alpha}{\partial x'} = \left[-\frac{x'}{2x_0^2} + \frac{x'}{x_0}\left(\frac{1}{x_0} - \frac{x'^2}{x_0^3}\right) - \frac{x'^2}{x_0^3\sqrt{2}}\exp(i\omega t - i\delta) + \frac{1}{\sqrt{2}}\left(\frac{1}{x_0} - \frac{x'^2}{x_0^3}\right)\exp(-i\omega t + i\delta)\right]\psi_0^*\psi_0$$

$$\langle p \rangle_\alpha = -i\hbar \int_{-\infty}^{\infty} \psi_\alpha^* \frac{\partial \psi_\alpha}{\partial x'} dx' = i\hbar \frac{1}{x_0^3\sqrt{2}}\exp(i\omega t - i\delta)\int_{-\infty}^{\infty} x'^2 \psi_0^*\psi_0 \, dx$$
$$- \frac{i\hbar}{\sqrt{2}}\exp(-i\omega t + i\delta)\int_{-\infty}^{\infty}\left(\frac{1}{x_0} - \frac{x'^2}{x_0^3}\right)\psi_0^*\psi_0 \, dx.$$

$$\langle p \rangle_\alpha = i\hbar \frac{1}{x_0\sqrt{2}}\left(\frac{1}{2}\exp(i\omega t - \delta i) - \frac{1}{2}\exp(-i\omega t + i\delta)\right) = -\frac{\hbar}{x_0\sqrt{2}}\sin(\omega t - \delta)$$

$$\begin{cases} \langle x \rangle_\alpha = \frac{1}{\sqrt{2}} x_0 \cos(\omega t - \delta) \\ \langle p \rangle_\alpha = -\frac{\hbar}{x_0\sqrt{2}}\sin(\omega t - \delta) \end{cases}$$

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b. 하이젠베르크 관점에서 계산해보아라.

→ 풀이. $a(t)$ 와 $a^\dagger(t)$ 를 구해서 $x^H(t)$ 와 $p^H(t)$ 를 구하는 게 일반적

$$\frac{d}{dt}x^H = \frac{1}{i\hbar}[x, H] = \frac{1}{i\hbar}\left[\frac{1}{2m}xp^2 + \frac{1}{2}m\omega^2x^3 - \frac{1}{2m}p^2x - \frac{1}{2}m\omega^2x^3\right]$$
$$= \frac{1}{i\hbar 2m}(xp^2 - p^2x) = \frac{1}{i\hbar 2m}\left(\cancel{p}[x,p] - pxp + [x,p]p + pxp\right)$$
$$= \frac{1}{i\hbar 2m}\cdot 2i\hbar p = \frac{p}{m}$$

$$\frac{d}{dt}p^H = \frac{1}{i\hbar}[p, H] = \frac{1}{i\hbar}\left[\frac{1}{2m}p^3 + \frac{1}{2}m\omega^2px^2 - \frac{1}{2m}p^3 - \frac{1}{2}m\omega^2x^2p\right]$$
$$= \frac{m\omega^2}{2i\hbar}(px^2 - x^2p) = \frac{m\omega^2}{2i\hbar}\left(x[p,x] - xpx + [p,x]x + xpx\right)$$
$$= \frac{m\omega^2}{2i\hbar}\cdot(-2i\hbar x) = -m\omega^2 x.$$

$$\begin{cases} \dfrac{d}{dt}x^H = \dfrac{p}{m} \\ \dfrac{d}{dt}p^H = -m\omega^2 x. \end{cases}$$

$$\dot{a}(t) = \sqrt{\frac{m\omega}{2\hbar}}\left(\frac{p}{m} + \frac{i}{m\omega}(-m\omega^2x)\right) = \sqrt{\frac{m\omega}{2\hbar}}\left(\frac{1}{m}p - i\omega x\right)$$
$$= -i\omega\sqrt{\frac{m\omega}{2\hbar}}\left(\frac{i}{m\omega}p + x\right) = -i\omega a$$

$\therefore \dot{a} = -i\omega a, \quad a(t) = e^{-i\omega t}a_0.$

$$\dot{a}^\dagger(t) = \sqrt{\frac{m\omega}{2\hbar}}\left(\frac{p}{m} - \frac{i}{m\omega}(-m\omega^2x)\right) = \sqrt{\frac{m\omega}{2\hbar}}\left(\frac{p}{m} + i\omega x\right)$$
$$= i\omega\sqrt{\frac{m\omega}{2\hbar}}\left(-\frac{i}{m\omega}p + x\right) = i\omega a^\dagger.$$

$\therefore \dot{a}^\dagger = i\omega a^\dagger, \quad a^\dagger(t) = e^{i\omega t}a_0^\dagger.$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) = \sqrt{\frac{\hbar}{2m\omega}}\left(e^{-i\omega t}a_0 + e^{i\omega t}a_0^\dagger\right)$$
$$p = i\sqrt{\tfrac{1}{2}m\hbar\omega}(a^\dagger - a) = i\sqrt{\tfrac{1}{2}m\hbar\omega}\left(e^{i\omega t}a_0^\dagger - e^{-i\omega t}a_0\right)$$
$$x_0 = \sqrt{\frac{\hbar}{2m\omega}}(a_0 + a_0^\dagger), \quad p_0 = i\sqrt{\tfrac{1}{2}m\hbar\omega}(a_0^\dagger - a_0)$$

$$\begin{cases} x(t) = x_0\cos\omega t + \dfrac{1}{m\omega}p_0\sin\omega t \\ p(t) = p_0\cos\omega t - m\omega x_0\sin\omega t. \end{cases}$$

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$$\langle \alpha | x(t) | \alpha \rangle = \cos\omega t \langle \alpha | x_0 | \alpha \rangle + \frac{1}{m\omega} \sin\omega t \langle \alpha | p_0 | \alpha \rangle$$

$$\langle \alpha | p(t) | \alpha \rangle = \cos\omega t \langle \alpha | p_0 | \alpha \rangle - m\omega \sin\omega t \langle \alpha | x_0 | \alpha \rangle$$

$$\langle 0 | x_0 | 0 \rangle = \int x' \psi_0^* \psi_0 \, dx' = 0 \quad , \quad \langle 1 | x_0 | 1 \rangle = \int x' \cdot \frac{2}{x_0^2} x'^2 \psi_0^* \psi_0 \, dx' = 0$$

$$\langle 0 | x_0 | 1 \rangle = \int x' \psi_0^* \psi_1 \, dx' = \frac{\sqrt{2}}{x_0} \int x'^2 \psi_0^* \psi_0 \, dx' = \frac{\sqrt{2}}{x_0} \cdot \frac{x_0^2}{2} = \frac{x_0}{\sqrt{2}}$$

$$\langle 1 | x_0 | 0 \rangle = \{ \langle 0 | x_0 | 1 \rangle \}^* = \frac{1}{\sqrt{2}} x_0$$

$$\therefore \langle \alpha | x_0 | \alpha \rangle = \frac{1}{2} [\langle 0 | x_0 | 0 \rangle + \langle 1 | x_0 | 1 \rangle + e^{i\delta} \langle 0 | x_0 | 1 \rangle + e^{-i\delta} \langle 1 | x_0 | 0 \rangle]$$

$$= \frac{x_0}{\sqrt{2}} \cdot \frac{1}{2} (e^{i\delta} + e^{-i\delta}) = \frac{x_0}{\sqrt{2}} \cos(\delta)$$

$$\langle 0 | p_0 | 0 \rangle = -i\hbar \int \psi_0^* \frac{\partial \psi_0}{\partial x'} \, dx' = i\hbar \int \frac{x'}{x_0^2} \psi_0^* \psi_0 \, dx' = 0$$

$$\langle 1 | p_0 | 1 \rangle = -i\hbar \int \psi_1^* \frac{\partial \psi_1}{\partial x'} \, dx' = -i\hbar \frac{2}{x_0^2} \int x' \left(1 - \frac{x'^2}{x_0^2}\right) \psi_0^* \psi_0 \, dx' = 0$$

$$\langle 0 | p_0 | 1 \rangle = -i\hbar \int \psi_0^* \frac{\partial \psi_1}{\partial x'} \, dx' = -i\hbar \frac{\sqrt{2}}{x_0} \int \left(1 - \frac{x'^2}{x_0^2}\right) \psi_0^* \psi_0 \, dx' = -i\hbar \frac{\sqrt{2}}{x_0} \left(1 - \frac{1}{x_0^2} \cdot \frac{x_0^2}{2}\right)$$

$$= -i\hbar \frac{\sqrt{2}}{x_0} \left(1 - \frac{1}{2}\right) = -i\hbar \frac{1}{x_0 \sqrt{2}}$$

$$\langle \alpha | p_0 | \alpha \rangle = \frac{1}{2} \left[e^{i\delta} \langle 0 | p_0 | 1 \rangle + e^{-i\delta} \langle 1 | p_0 | 0 \rangle \right] = -i\hbar \frac{1}{x_0 \sqrt{2}} \cdot \frac{1}{2} (e^{i\delta} - e^{-i\delta})$$

$$= -i\hbar \frac{1}{x_0 \sqrt{2}} \cdot i \sin(\delta) = \frac{\hbar}{x_0 \sqrt{2}} \sin(\delta)$$

$x_0 = \sqrt{\frac{\hbar}{m\omega}}$

$$\frac{1}{m\omega} \langle \alpha | p_0 | \alpha \rangle = \frac{\hbar}{\sqrt{2}} \cdot \sqrt{\frac{m\omega}{\hbar}} \cdot \frac{1}{m\omega} \sin(\delta) = \frac{x_0}{\sqrt{2}} \sin(\delta)$$

$$m\omega \langle \alpha | x_0 | \alpha \rangle = \frac{\hbar}{x_0 \sqrt{2}} \cos(\delta)$$

$$\langle \alpha | x(t) | \alpha \rangle = \frac{x_0}{\sqrt{2}} \cos\delta \cos\omega t + \frac{x_0}{\sqrt{2}} \sin\delta \sin\omega t = \frac{x_0}{\sqrt{2}} \cos(\omega t - \delta)$$

$$\langle \alpha | p(t) | \alpha \rangle = \frac{\hbar}{x_0 \sqrt{2}} \sin\delta \cos\omega t - \frac{\hbar}{x_0 \sqrt{2}} \cos\delta \sin\omega t$$

$$= -\frac{\hbar}{x_0 \sqrt{2}} \sin(\omega t - \delta)$$

$$\begin{cases} \langle x \rangle_\alpha (t) = \frac{x_0}{\sqrt{2}} \cos(\omega t - \delta) \\ \langle p \rangle_\alpha (t) = -\frac{\hbar}{x_0 \sqrt{2}} \sin(\omega t - \delta) \end{cases}$$

슈뢰딩거 관점으로 푼 것과 결과가 같다.